x = ?(**Problem 2.1 in** [**1**]).
https://www.linkedin.com/groups/8313943/8313943-6385054229752528898
Find all integers x which satisfy the equation
cos ((π/8)(3x - √(9x² + 160x + 800)) = 1.
Solution by Arkady Alt , **San Jose**, **California**, **USA**.
Since $\cos\left(\frac{\pi}{8}\left(3x-\sqrt{9x^2+160x+800}\right)\right) = 1 \iff \frac{\pi}{8}\left(3x-\sqrt{9x^2+160x+800}\right) = 2n\pi \iff$
$\frac{3x-\sqrt{9x^2+160x+800}}{16} = n, n \in \mathbb{Z}$ we have to solve in integers equation
$3x-\sqrt{9x^2+160x+800} = 16n \iff 9x^2+160x+800 = (3x-16n)^2 \iff$
$5x+3nx-8n^2+25 = 0 \iff x = \frac{8n^2-25}{3n+5}$.
Since $\gcd(8n^2-25, 3n+5) = \gcd(n^2+15n+25, 3n+5) = \gcd(3n^2+45n+75, 3n+5) =$
$\gcd(40n+75, 3n+5) = \gcd(n+10, 3n+5) = \gcd(n+10, 25) \in \{1, 5, 25\}$.
Since x is integer iff $\gcd(n+10, 25) = |3n+5|$ then possible three options:
$|3n+5| = 1 \iff n = -2, |3n+5| = 5 \iff n = 0$ and $|3n+5| = 25 \iff n = -10$.
Hence for $n = -2$ we obtain $x = \frac{8\cdot 4-25}{-6+5} = -7,$ for $n = 0$ we obtain $x = -5$
and for $n = -6$ we obtain $x = \frac{8\cdot 100-25}{3\cdot(-10)+5} = -31$.
So, $\frac{3x-\sqrt{9x^2+160x+800}}{16}$ is integer only for $x = -5, -7, -31$.
1. **Arkady Alt**, Math Olympiads Training-Problems and solutions.
(This book is a translated into English extended and significantly added version of author’s brochures "Guidelines for teachers of mathematics to prepare students for mathematical competitions" published at 1988 in Odessa).